Fabry Perot Interferometer

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Contents

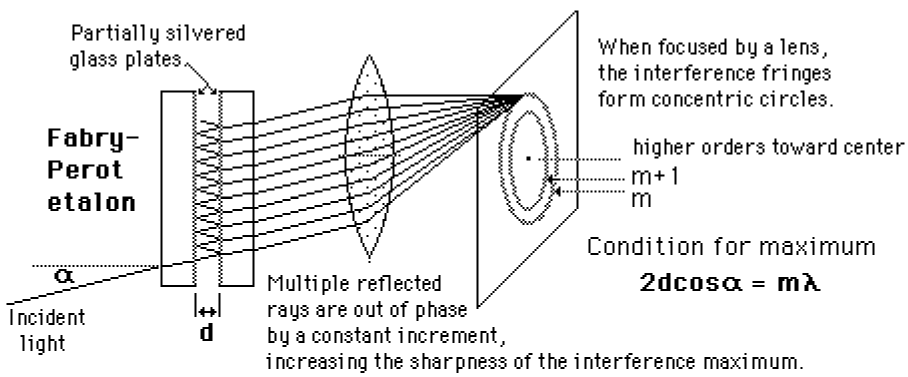
1 AIM

1.To find wavelength of a monochromatic light.
2.To Determination of gap between the plates of fabry-perot etalon from the fringe pattern at different micrometer readings.
3.To find the finesse and free spectral range (FSR) of etalon from the fringe calibration at different cavity thickness.

2 Apparatus required

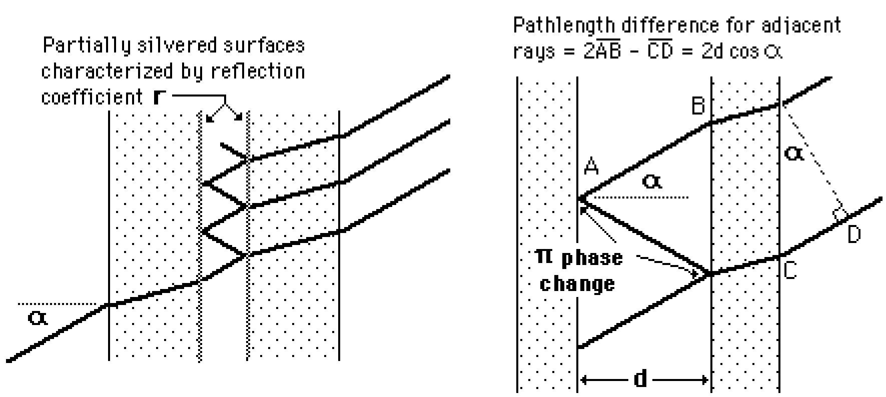
3 Introduction

This interferometer makes use of multiple reflections between two closely spaced partially silvered surfaces. Part of the light is transmitted each time the light reaches the second surface, resulting in multiple offset beams which can interfere with each other. The large number of interfering rays produces an interferometer with extremely high resolution, somewhat like the multiple slits of a diffraction grating increase its resolution.



4 Theory

The Fabry-Perot Interferometer makes use of multiple reflections which follow the interference condition for thin films. The net phase change is zero for two adjacent rays, so the condition $2dcos\theta = \lambda$ represents an intensity maximum.



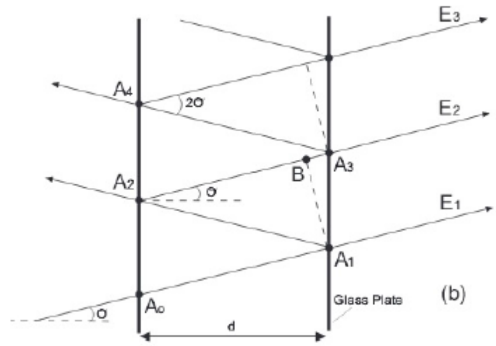


Figure1. Fabry - Perot cavity

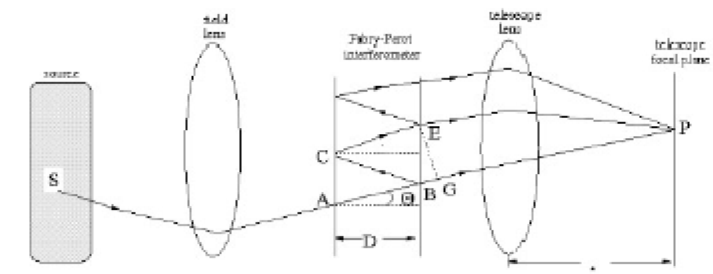


Figure2.Geometrical optics of the Fabry-Perot interferometer.

Consider a ray of light incident at an angle θ. The Path difference p, between two beams can be written as

$$p = A_1A_2 + A_2B$$

From the figure we can write,

$$A_2B = A_1A_2cos\theta$$

from eqn 1 & we get,

$$p = A_1A_2(1 + cos2\theta)$$

$$p = A_1A_2(2cos^2\theta)$$

$$p = 2dcos\theta$$

Where $d = A_1A_2cos\theta$. If the cavity has a medium with refractive index n, the path difference between consecutively reflected beams is,

$$p = 2ndcos\theta$$

Where D is the separation, n is the refractive index of the medium between plates and θ is the angle of the reflections between plates. Since all the multiple re ections come out parallel to each other, a lens can be used to combine the beams and observe the interference pattern. The transmission coefficient (t) is the fraction of the incident electric field that is transmitted and the reflection coefficient (r) is the fraction that is reflected. The transmission intensity of the emerging light beams(E1;E2 & E3) can be written as.

$$E_1 = E_o t t^{'} e^{i\omega t}$$

$$E_2 = E_o t r r t^{'} e^{i\omega t - \delta}$$

$$E_2 = E_o t r^4 t^{'} e^{i\omega t - 2\delta}$$

therefore,

$$E_N = E_o t r^{2(N-1)} t^{'} e^{i(\omega t - (N-1)\delta)}$$

E_o is the amplitude of the incident electric field. The exponential shows that electric filed is time dependent and δ is the phase difference between adjacently transmitted beams. The total electric field is,

$$E_t otal = a + aR^{'} + aR^{'2} + ... = \frac{a}{1-R^{'}}$$

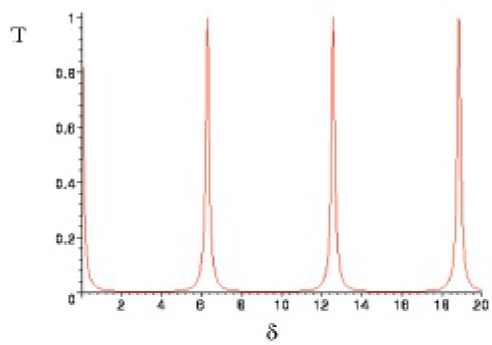


Figure3.Transmission as a function of phase,r=0.95

Where,$a = E_o t t^{'} e^{i\omega t}$ and $R^{'} = r^2 e^{-i\theta}$ Therefore the total transmitted electric field is,

$$E_{transmittedtotal} = \frac{E_o t t^{'} e^{i\omega t}}{1 - r^2 e^{-i\delta}}$$

The Intensity of the transmitted electric field is give by, $I_{transmitted} = E_{total} E^*_{total}$

$$I_{transmitted} = \frac{I_o}{(1+r^4) - 2r^2 cos\delta}$$

here I_0 is the intensity of the incident electric field. Using the identity $cos\delta = 1 - sin^2(\frac{\delta}{2})$ and $tt^{'} + r^2 = 1$ the transmitted intensity becomes,

$$I_{transmitted} = \frac{I_o}{(1 + F sin^2 \frac{\delta}{2})}$$

where $F = \frac{4R}{(1-r)^2}$ and $R = r^2$. The transmittance is given by the equation,

$$T = \frac{1}{1 + f sin^2 \frac{\delta}{2}}$$

where,

$$\delta = 2\pi \frac{p}{\lambda}$$

Constructive interference will occur when the phase difference between adjacent beams is an integer multiplied by 2π. From the above Eq we see that path difference p must be integral number of wavelengths. This situation is similar to a cavity in which standing waves occur when the length of the cavity is integral numbers of wavelengths so that the wave will be in phase and result in constructive interference. The transmittance of the interferometer is plotted as a function of phase difference δ in the fig. Note that δ is proportional to the path difference p. Note that the transmission spectrum consists of number of discrete peaks. The peaks correspond to the plate spacing corresponding to an integral number of half wavelengths that ft in within the Fabry-Perot cavity.

4.1 Finesse

Finesse is related to the resolving power called the Finesse. It is defined as the ratio of the separation of adjacent maxima over the FWHM. Therefore using below equations and considering 2π as the separation of adjacent peaks (which can vary by changing d) we get,

$$f = \frac{2\pi}{4/\sqrt{F}} = \frac{\pi\sqrt{F}}{2}$$

4.2 Free Spectral Range

The free spectral range tells us the range of wavelengths that can be observed. From figure2, you can see that the interference pattern of the interferometer is repeated as the path difference changes. The transmission of the interferometer has a maximum when the path difference between consecutive reflections is an integer number of wavelengths. The properties of the interferometer in terms of the standing waves in the cavity. For these standing waves, a node must occur at the ends of the cavity (see figure 4). There are two ways the number of nodes in the standing waves in the cavity can change. If the wavelength of light changes, a different number of nodes will be accommodated within the cavity. If, however, the wavelength remains constant the cavity length itself must change until one more (or less) node is accommodated. FSR can be defined as change in wavelength between maxima in the transmittance. If the cavity length remains constant, a free spectral range is the difference in wavelength between adjacent modes that give you constructive interference. In fig 4 these correspond to $\lambda 2$ and $\lambda 3$. Since the cavity length is the same, we have,

$$m\lambda_2 = (m+1)\lambda_3$$

The difference between there two wavelengths is FSR

$$\lambda_{fsr} = \lambda_2 - \lambda_3 = \frac{\lambda}{m}$$

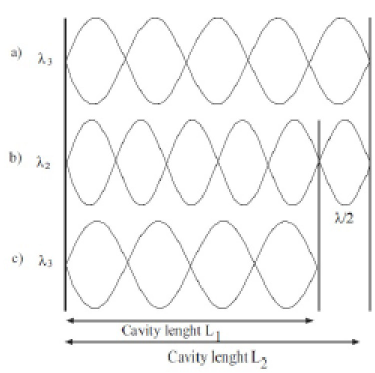


Figure4.Change of one FSR

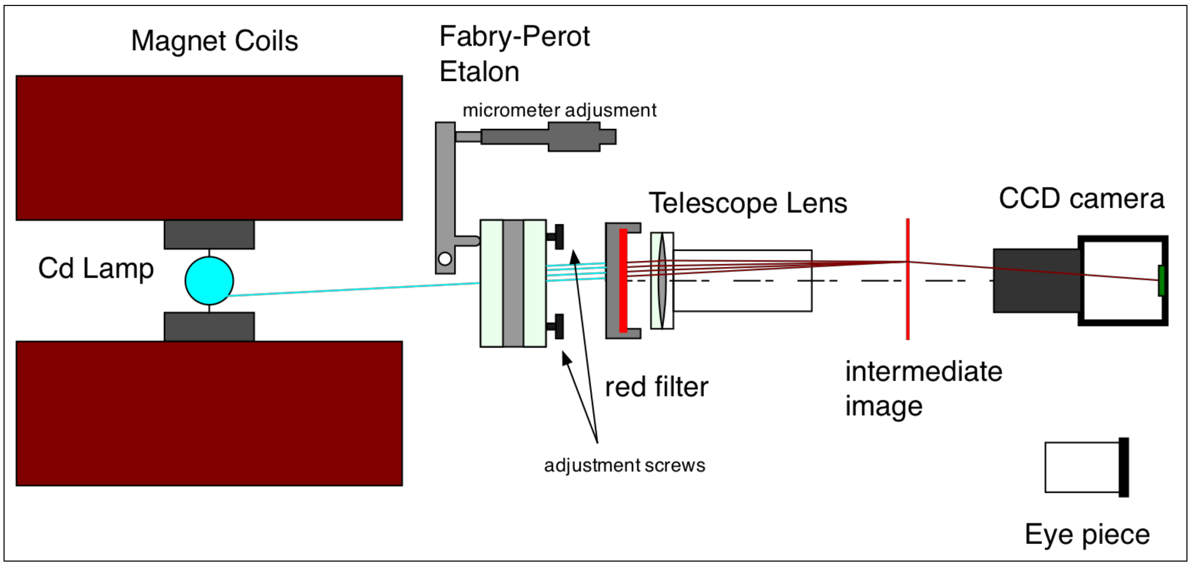
Here λ_1 andλ_2 are very close to each other and m is the index of the cavity mode or the number of half-wavelengths that fit in the cavity.

$$m = \frac{2d}{\lambda}$$

From the above two equations we can write

$$\lambda_{fsr} = \frac{\lambda^2}{2d}$$

5 Experimental Setup



6 Observations and Calculations

6.1 Calibrating the micrometer

For more accurate measurement of the mirror movement we can calibrate the micrometer as follows. Now turn the micrometer upto count of 20 fringes or more. Note the changes in micrometer reading and record this value as $d^{'}$. But the actual mirror movement is given by $d = \frac{N\lambda}{2}$ where λ is the wavelength of the light source and N is the no of fringes counted.Then $\Delta = \frac{d}{d'}$ is the calibration constant for the micrometer. Here N=20,$d^{'} = 0.0858mm$ $\lambda = 532nm$ & least count of the screw Gauge=0.01mm

$$d = \frac{N\lambda}{2} = \frac{20*532*10^{-9}}{2}m = 5.32*10^{-6}m$$

$$\Delta = \frac{d}{d'} = \frac{5.32*10^{-6}m}{85.806*10^{-6}m} = 0.062$$

6.2 To find λ

(N)no. of fringes	Initial reading(A)(in mm)	Final reading(B)(in mm)	(A-B)(in mm)	Avg(d) (in mm)
20	1.04	1.18	0.14	
20	1.18	1.29	0.11	0.1066
20	1.29	1.36	0.07	

now we have put all the values in the formula given

$$\lambda = \frac{2d_{mean}}{N} * \Delta = \frac{2*0.1066*10^{-3}}{20} * 0.062 = 660.92nm$$

6.3 To find spacing of etalon

distance between screen and etalon,D=45cm and λ=650 nm and m= Order of the circular fringe

fringe No	Radius(cm)	$R^2(cm^2)$	$X_m^2 = X_{n+m}^2 - X_n^2$	order m	t=$\frac{mD^2\lambda}{X_n^2}(m)$	Avg t in mm
1	0	0	0	0	0	
2	0.54	0.291	0.291	1	4.523*10^{-3}	
3	0.69	0.476	0.476	2	$5.53 * 10^{-3}$	
4	0.92	0.846	0.846	3	$4.66 * 10^{-3}$	4.33
5	1.01	1.02	1.02	4	$5.06 * 10^{-3}$	
6	1.15	1.322	1.15	5	$5.72 * 10^{-3}$	
7	1.28	1.638	1.638	6	$4.82 * 10^{-3}$	
8	1.45	2.10	2.10	7	$4.38 * 10^{-3}$	

7 Error Analysis

The actual wavelength for the RED laser is 650 nm My experimental value came as = 686 nm
so error=$\Delta\lambda = \lambda_0 - \lambda = 660.92 - 650 = 10.92nm$
Percentage in Error=$\frac{\Delta\lambda}{\lambda_0} * 100 = \frac{10.92}{650} * 100 = 1.68\%$

8 Results

We have found the wavelength of the Red Laser to be 660.92 nm with 1.68 % error and the spacing between the Etalon is found to be 4.33mm.

9 Precautions

1. Do not touch or contact in any way either the front or back surfaces of the mirror pieces. Doing so will permanently damage the mirror coatings.
2. Avoid eye exposure to the direct laser beam.
3. Move the micrometer screw very slowly

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10 References

1. Optics by Basudev Ghosh
2. Optics by Ajay Ghatak
3. www.google.com
4. Lab Manual
5. wikipedia